Geomechanics

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Abstract

This article discusses special challenges posed for solid mechanics by problems in geotechnology and geophysics and recent advances made by applications of solid mechanics to these areas. In addition, the article discusses selected promising areas of current research, focussing on results and needs for further research in the following areas: multi-axial constitutive relations; the coupling of mechanical response with fluid flow and chemistry; fracture growth, interaction and network development; and earthquake dynamics. © 1999 Published by Elsevier Science Ltd. All rights reserved.

1. Introduction

Many important problems facing the nation involve significant components of geomechanics: geologic disposal of radioactive waste, terrestrial sequestration of carbon dioxide to mitigate adverse effects on the atmosphere, efficient underground storage of natural gas, discovery and recovery of hydrocarbons, penetration of earth structures for military applications, and earthquake and volcano hazard mitigation. Inadequate knowledge of the mechanics of geomaterials can be expensive and dangerous. One prominent example is the AMOCO (Standard Oil) building in Chicago: The three by four foot slabs of imported Italian Carrara marble that formed the building facade began to crack in the 1980's, apparently from thermal cycling and, possibly, environmental effects; because of safety concerns, the 43,000 slabs were replaced at a cost of $70 million, about half the original cost of erecting the building two decades earlier.

Historically, geologic materials and problems have provided fertile territory for exploration by luminaries in solid mechanics. Examples include the seminal investigations of the failure of sandstone by Coulomb, experiments on the pressure dependence of the strength of Carrara marble by von Karman,

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investigations of the earth tides and the isostatic support of the continents and mountains by A.E.H. Love, and studies of fold formation and waves in fluid-saturated porous materials by Biot.

Several recent technological and political developments have made the present particularly propitious for research in the mechanics of geological materials. The global positioning system (GPS) and synthetic aperture radar (SAR) are providing an unprecedented ability to monitor deformations of the earth's crust. Although entangled with political and societal questions, efforts in the United States and other countries to study geologic repositories for the safe disposal of nuclear wastes have provided impetus for understanding the mechanical behavior and transport properties of crustal materials. The desire to sequester carbon dioxide in order to meet the requirements of the Kyoto accords on global climate change may provide further impetus in the coming decades. As in other areas of mechanics, increases in computational power have made possible the assimilation and interpretation of large amounts of data and the exploration of models that involve more complex loading, geometries and material behaviors. At the same time, this increased computational power imposes a need for insightful analysis to interpret computer solutions, laboratory experiments to provide constraints on material behavior, and field measurements to test and verify models.

Geological systems have a variety of complex characteristics atypical of other engineering systems: They are strongly heterogeneous; they have discontinuities on a variety of length scales; the material response is pressure-sensitive and coupled to thermal, transport and chemical processes; because rock masses can be accessed directly only at the surface or through tunnels and boreholes, uncertainties about material and transport properties are inevitable; properties evolve with time and, at best, can only be modified by human intervention.

Another inherent difficulty in geomechanics is the enormous range of time scales. At the shorter end are time scales similar to those in other engineering problems: wave propagation or penetration problems of milliseconds, earthquake occurrences and cutting operations of seconds to minutes, drilling for days or weeks, seasonal storage of natural gas storage. At the upper end are, however, time scales far longer. Time intervals between occurrences of major earthquakes at a particular location may range from hundreds to thousands of years, terrestrial sequestration of carbon dioxide should prevent the gas from returning to the atmosphere for hundreds of years, and requirements for licensing of nuclear waste repositories in the United States mandate that the waste be isolated from the accessible environment for 10,000 years.

Length scales are similarly disparate. The behavior of different rock types, say, sandstone versus marble, depends strongly on microstructural features on the size scale of microns to millimeters: the mineral components, shape and size of grains, cement, nature and distribution of porosity. Yet, geologic structures, such as hydrocarbon reservoirs or aquifers, and man-made structures, such as tunnels, underground transportation systems, or the proposed high-level nuclear waste repository at Yucca Mountain, Nevada, extend tens or hundreds of kilometers. Similarly, a variety of experimental, theoretical and computational evidence has shown that the stability of slip on rock surfaces is strongly dependent on a characteristic distance that enters the description of the frictional behavior. Laboratory measurements suggest this distance is in the range of 1 to 100 μm, and, though there are indications that the distance may be larger, up to 500 mm, on faults in the Earth's crust, it is orders of magnitude smaller than the tens or hundreds of kilometer rupture lengths of large earthquakes.

Another aspect in which geomechanics differs from other areas of mechanics is the role of experiments. For typical structural materials, say, steel, the relevant properties in an application, e.g., a beam in a large structure (of length on the order of 20 feet), can reliably be determined from laboratory testing on a small (several inches in length) specimen. In contrast, laboratory experiments that duplicate natural conditions are, generally, not feasible. The behavior of rock structures and the crust is strongly affected, often dominated, by the presence of large discontinuities, faults, joints, fractures and shear zones, that cannot be incorporated into laboratory size specimens. Attempts to determine the geometries

and properties of these structures by testing portions of them, for instance, a laboratory size piece of rock with a fracture, are ambiguous because the behavior of these features depends on the topography at different length scales. The problem is exacerbated by the interaction of fluid flow and chemistry with deformation, as is typically the case in the Earth's crust. Despite the limitations of experiments in this context, they remain essential for identifying mechanisms and for determining constitutive properties of rock material and surfaces. The limitations of both time and length scales do, however, accentuate the need for careful analysis and rigorous computations to extrapolate results to *in situ* conditions.

Ultimately, field observations are required to understand geomechanical systems and to test predictions based on laboratory measurements, theoretical modeling and computational simulations. Unfortunately, field studies are generally difficult, expensive and time-consuming. To be successful they require appropriate background of theoretical, laboratory and computational studies. Even then, there are many examples of surprises provided by field measurements despite predictions based on the best current knowledge: the nonoccurrence (yet) of the Parkfield earthquake after a decade-long effort of prediction and monitoring; measurements, about 10 years ago, of downhole pressures much larger than those predicted by models of hydraulic fracturing then current in the petroleum industry; and abundant fluids in boreholes at depths, nearly 10 km, where all pore space was thought to be closed.

As suggested by the discussion above, geomechanics is far too broad an area for which to summarize recent research in an article of this length. What follows is a discussion of selected areas of geomechanics and some suggestions for needed research.

2. Multi-axial constitutive relations

The vast majority of rock testing has used the axisymmetric compression configuration. Although there are conceptual and practical reasons for this choice, deformation states in applications are seldom limited to axisymmetry. Furthermore, in the most common testing configuration, constant lateral confining stress, the mean and shear stress (first and second invariants) change in fixed proportion. In contrast to structural metals, the inelastic behavior in most geomaterials depends on the mean stress. In data from conventional axisymmetric compression tests, separation of the effects of mean and shear stress is difficult, although the problem can be avoided in modern programmable testing apparatus.

A further drawback of the axisymmetric compression test is that two of the principal stresses are equal. Classical experiments indicated the effect of the intermediate principal stress on the qualitative behavior. Although there have been some further experiments of this type, they have been inadequate to constrain the behavior for applications. In the past, investigation of more complex deformation states and stress paths may have been superfluous because such information could not be incorporated into applications, but this is no longer the case. Advances in multi-axial experiments to provide input for and to test more elaborate constitutive models in geomechanics have not kept pace with dramatic advances in computing power.

Research over the past 20 years in mechanics has demonstrated that details of the constitutive behavior, in particular, the response to abrupt changes in the pattern of deformation, can dramatically affect predictions of shear localization. Since zones of localized deformation (faults and fractures) control, to large extent, the mechanical and transport properties of the crust, understanding their formation and evolution remains a critical issue. An example of how these details can affect the development of major geologic structures is recent work on instabilities in layered systems. This work demonstrates that predictions of the relative onsets of faulting and folding depend strongly on whether the layer is modeled by a flow or deformation type of elastic-plastic constitutive relation. Although there are reasons to think that the deformation theory is a more accurate representation in this case, the

values of the relevant moduli entering the theory are largely unconstrained by experimental observations or by predictions based on explicit assumptions about the micromechanical mechanisms.

Another example of the significance of details of the constitutive response for applications is provided by studies in connection with the Waste Isolation Pilot Plant (WIPP) in New Mexico. This site was granted a U.S. Federal license in May, 1998 to begin burying plutonium and will be the world's first deep underground nuclear waste storage facility. Although opponents continue to dispute the safety of the site, the magnitude and sophistication of the modeling is probably unprecedented and must be regarded as an important success for geomechanics. This modeling was based on a combination of micromechanical studies of deformation and damage mechanisms, laboratory testing, computational simulations and field verification. A Mises flow potential was assumed initially, but comparison of predictions with field measurements appeared to indicate better agreement with a Tresca flow potential.

A further impediment to a better understanding of constitutive relations for rocks is the complexity of microscale processes. In contrast to the inelastic deformation of metals, for which dislocation motion and slip on crystal slip systems are the underlying mechanisms, the inelastic deformation of rocks may result from inter- and intra-granular microcracking, kinking, twinning, pore collapse, crushing of cement between hard particles, in addition to dislocation motion. The different constituents of rocks, say, quartz versus feldspar in granites, may deform by different mechanisms and exhibit different dependencies on pressure and temperature. Microcracking in preferred directions causes anisotropy of mechanical and transport properties to evolve with loading. Recent interest has focussed on weak, highly porous rocks that exhibit characteristics of both hard rock and granular aggregates.

New technologies, such as laser scanning confocal microscopy, and more sophisticated application of others, such as monitoring of acoustic emissions, have made possible increasingly detailed descriptions of microscale deformation mechanisms and three-dimensional pore geometries. In addition, there has been much effort in modeling the growth and interaction of microcracks in compressive stress fields, both from the point-of-view of fracture mechanics and from statistical interactions. Still, understanding of how microscale processes affect macroscale behavior is incomplete. In particular, there has been relatively little modeling of the effects of micromechanisms other than crack-growth on macroscopic constitutive relations.

2.1. Research needs

- Multi-axial experiments to constrain better the dependence of material response on stress state and path.
- Better micromechanical models to predict the dependence of material response on stress state and path.
- Better understanding of how different microstuctural mechanisms of deformation affect response at larger length scales

3. Coupling of mechanical response with fluid flow and chemistry

Deformation of the crust is generally coupled with the flow of fluids and chemical interactions. Although the interaction of the deformation, flow, and chemistry is often neglected, primarily for simplicity, recent observations, experiments and modeling have demonstrated its importance in a variety of problems. For example, fractures provide major conduits for the flow of fluid in the crust and the aperture is affected not only by tractions applied to the fracture but also by chemical deposition or dissolution, which, in turn, depends on the rate of fluid flow through the fracture. Conversely, fluid flow and chemical action can alter the stresses transmitted across the fractures. Laboratory results have

suggested that porosity decrease and chemical deposition enhanced by temperatures at mid-crustal depths can modulate the occurrence of large earthquakes. There is evidence that stress-assisted corrosion cracking influences the long-term strength of the crust, but understanding has not progressed to the point where reliable predictions can be made for applications.

Poroelasticity, first formulated in a complete fashion by Biot, provides a minimal framework for including the effects of coupling between deformation and fluid diffusion. This formulation has seen increasing application to problems in the petroleum industry and problems of crustal deformation. Unfortunately, even within this idealized treatment, the appropriate material parameters are often uncertain. They are difficult to measure in the laboratory and their inference from field observations is not straightforward. In addition, the treatment does not include chemical effects and, because it is linearized, does not include effects of stress on transport properties, e.g., permeability, as just discussed for fracture aperture changes.

The mechanical effect of pore fluid is encapsulated in the effective stress principle: the effect of the pore fluid is included by replacing the mean stress with a linear combination of the mean stress and pore pressure. The relation of the coefficients in this combination to the constituents or to the type of deformation mechanisms, say, microcracking versus shear-induced compaction, is not clear. Nor is the effect of stress path and history. Thermal effects on the pore fluid can also be significant in a variety of applications: stability of well-bores in the petroleum industry, initiation and propagation of earthquakes at midcrustal depths and nuclear waste disposal. Again a linearized treatment provides a minimal framework, but inelastic, history-dependent effects are often significant and poorly understood.

A familiar and important example of the coupling between deformation and fluid flow is hydraulic fracturing: driving of a fracture in the crust by pressurized fluid. This technique has been widely used in the oil and gas industry to enhance the recovery of hydrocarbons from underground reservoirs and by geophysicists to measure stress in the crust. Hydraulic fracturing is also a mechanism in the formation of dykes and the transport of magma through the crust. The basic theory of hydraulic fracturing, established nearly 40 years ago, is based on earlier elasticity solutions. Since that time, much effort has been devoted to the mathematical and computational modeling of the phenomenon. The problem is notoriously difficult because of the non-linear coupling of the flow of fluid through the crack, possibly with leak-off into the adjacent solid, and the propagation of the fracture. Despite the amount of attention devoted to the subject, a thorough understanding of it has been elusive and the ability to predict with confidence the results of field operations has remained poor.

Considerable progress has been made recently in characterizing the nature of the stress and fluid pressure fields near the tip of a propagating fracture and identifying the dominating terms in different flow regimes. The near-tip region is important because this is where the actual processes of material breakdown occur and this region holds the key to understanding different propagation regimes. This work has the promise of leading to a more accurate and efficient representation of the fracturing process and better simulations of field treatments.

Other examples of applications in which this coupling of deformation, flow and chemistry is essential are the sequestration of carbon dioxide in reservoirs or aquifers and the nucleation of earthquakes at mid-crustal depths due to the formation and rupture of permeability seals. Depending on the chemistry of the fluid, rock type, and fluid flow rates, deposition may enhance seals and degrade permeability or dissolution may have the opposite effect.

3.1. Research needs

- Better understanding of the effects of stress path and history on the effective stress principle.
- Better understanding of how the macroscopic effect of pore fluid is related to microscale mechanisms of deformation.

- Experiments to illuminate the role of coupling between chemistry, stress, temperature, and fluid flow.
- Models of this coupling that are sufficiently simple to be constrained by laboratory observations and allow field predictions.
- Better understanding of conditions leading to different regimes of hydraulic fracture propagation, especially in porous permeable rocks.

4. Fracture growth, interaction and network development

The last 20 years have seen wide application of fracture mechanics to geotechnical and geomechanical problems. Although a reasonable understanding has been achieved of how discrete fractures develop and grow, fractures in geological materials seldom occur in isolation. Because the applied principal stresses are almost always compressive, the tensile stresses driving opening cracks are necessarily local and, hence, crack growth is, at least initially, stable. A critical issue is how fractures propagate, interact, and link-up to form fracture networks. Such networks not only control the mechanical properties but are generally the principal pathways for fluid flow. Multiple, segmented fractures have also been increasingly recognized to play an important role in the hydraulic fracturing process.

The ability to model the formation of fracture networks is inhibited by (at least) two aspects: (i) Three-dimensional geometry is clearly important in establishing flow paths, but current understanding of this aspect is rudimentary; (ii) the understanding of how inelasticity (damage) associated with fracture affects conditions and directions for propagation and how the permeability structure surrounding a primary fracture affects the hydrology is limited.

The mechanics of fracture propagation can be largely understood in terms of two dimensional models. In particular, for a fracture with a smoothly turning edge in a linear-elastic material the asymptotic near-tip field decomposes exactly into separate plane and anti-plane two dimensional problems. Three-dimensional effects are, however, critical to understanding the extension and link-up of fractures to form hydrological networks: an asperity will completely block flow in a two-dimensional model, but in three dimensions flow can simply go around this impediment.

The most prevalent models of fracture in geomaterials are those that treat the crack-tip as a singularity of stress in an otherwise linear elastic body or cohesive zone models in which the inelasticity is confined to a line ahead of the crack. Although these are sometimes adequate models, the inelasticity and damage in the vicinity of the crack-tip can dramatically alter the out-of-plane growth, the link-up and interaction of cracks and the formation of hydrologic pathways. The history of stress near a crack-tip can be complex. Consequently, better understanding of this issue is linked to the availability of appropriate multi-axial constitutive relations for inelastic deformation.

4.1. Research needs

- Better understanding of crack growth and interaction under conditions of overall compression and their relation to hydrologic properties.
- Better understanding of the inelasticity accompanying fracture growth in geological materials under overall compression.

5. Earthquake dynamics

Prompted by the Earthquake Hazards Reduction Program and dramatic increases in the quantity and

quality of seismological and crustal deformation data, understanding of the mechanics of earthquakes has progressed rapidly in the last 20 years. Although a goal of earthquake prediction does not appear achievable in the near future (or ever, some would argue), understanding of the relation of the earthquake instability to material properties and long-term tectonic strain-rates has advanced considerably. In addition, mechanics has played in a key role in integrating geologic observations of faults, seismological measurements of deformation due to waves radiated from earthquakes and laboratory experiments on failure and friction.

An important development in earthquake mechanics has been the formulation of the rate and state dependent constitutive relation for frictional slip. Simply put, the coefficient of friction depends on the current rate of slip and the past history, as reflected in a dependence on the state. This framework was first identified in experiments and has proven remarkably successful in interpreting and predicting a variety of modes of stable and unstable slip in laboratory experiments. When used as a condition on fault surfaces in continuum mechanics models of crustal scale deformation, rate and state dependent friction successfully reproduces a number of observed earthquake features, including repeated events on fault segments and the depth dependence of slip.

Although recent experiments on plexiglass have provided insight into the relation of the dependence on state to the evolving topography of the surface and the time-dependent deformation of contact points, understanding of the mechanics underlying the rate and state laws remains incomplete. This lack of understanding causes uncertainty in the appropriate values of parameters, in particular a characteristic length scale, for in situ conditions. As measured in the laboratory, this length scale is quite small, typically 1 to 100 µm. Appropriate values for the crust are uncertain. If they are as small as measured in the laboratory, this poses a stringent condition on discretization lengths needed for accurate numerical modeling. In addition, further information is needed about the temperature dependence of these relations and the behavior at high rates of slip, comparable to those during earthquakes.

Recent research has focussed on the use of the rate and state dependent frictional constitutive relation in models of multiple event initiation, earthquake clustering and aftershocks, and models of the dynamics of the earthquake event. A particular issue in the latter area has been that the duration of earthquake slip at a point on the fault surface, as inferred from seismological observations, is much less than the total rupture time, as expected from models of dynamic crack growth. Possible resolutions of this discrepancy include the effects of strong heterogeneity, dissimilarity of elastic properties across a fault, and certain types of the rate and state dependent friction relations. More specifically, short-duration slip pulses are not possible for relations in which the friction stress is a decreasing function of velocity and does not depend at all on state, and for relations that do not allow for restrengthening in purely stationary contact. Experimental or field data that would distinguish among these possible behaviors is limited.

Another issue is the underlying source of slip complexity, that is, the occurrence of slip events of different sizes with a distribution described by the phenomenological Gutenberg-Richter relation. In some instances, complexity is a consequence of the inherent nonlinear dynamics of frictional systems with state and rate dependent constitutive relations. Others maintain, however, that the complexity in these models is a result of certain idealizations and discretization and that the observed complexity reflects the inhomogeneity of the crust rather than the intrinsic dynamics of the system. An impediment to final resolution of this issue has been the difficulty and computational resources required for fully dynamic continuum solutions with complex frictional constitutive relations. This work in dynamic fracture of highly heterogenous systems is also relevant, at much smaller length scales, to heterogeneous technological materials, such as composites. A fundamental result that has emerged from this work is the identification of a new type of elastic wave, a persistent wave generated by the response of the propagating crack to local heterogeneities.

5.1. Research needs

- Experiments to constrain constitutive relations for frictional slip at high slip rates and at elevated temperatures.
- Better understanding of the physics of rate and state dependent slip and factors controlling the characteristic length that appears in phenomenological relations.
- Better and more efficient methods of modeling dynamic and heterogeneous slip.

Acknowledgements

Suggested reading

A.1. News items and popular science

Accounts of the Chicago AMOCO building and the planned opening of the WIPP appeared in the *The New York Times*: 'Gigantic face lift (headache to match)' by I. Wilkerson, April 2, 1988 and 'New Mexico site is approved for storage of nuclear waste' by M. L. Wald, May 14, p. 1, 1998. A non-technical summary of investigations of the WIPP site appears in Geotimes ('Waste Isolation Pilot Plant,' Weart, W. D., Rempe, N. T. and Powers, D. W., October, 1998.) Articles in *Science* by R. A. Kerr describe the Parkfield earthquake prediction experiment ('Stalking the next Parkfield earthquake,' **223**, 36–38, 1988; 'Quake prediction under way in earnest,' **233**, 520, 1986; 'Taking the pulse of Parkfield,' **236**, 145, 1987; 'Seismologists issue a no-win earthquake warning,' **258**, 742–743, 1992; 'Parkfield earthquakes skip a beat,' **259**, 1120–1122, 1993), the unexpected presence of water in a deep borehole ('German super-deep hole hits bottom,' **261**, 545, 1994), and synthetic aperture radar ('Watching the earth move', **272**, 1870, 1996). A summary of the scientific implications of the Kyoto climate change treaty and planned attempts to sequester carbon dioxide appears in *Science* articles by J. Kaiser and K. Schmidt (**281**, 504–506, 1998). Recent *Scientific American* articles describe use of the global positioning system (GPS) (T. A. Herring, 'The global positioning system,' **274**, 44–50, 1996) and satellite radar (D. Massonnet, 'Satellite radar interferometry,' **276**, 46–53, 1997) to measure earth movements. Another recent review article is 'GPS applications for geodynamics and earthquake studies' by P. Segall and J. L. Davis in *Annual Review of Earth and Planetary Sciences*, **25**, 301–336, 1997.

A.2. Books

Fundamentals of Rock Mechanics by J. C. Jaeger and N. G. W. Cook (2nd edition, Chapman and Hall, London, 1976) provides a technical introduction to the subject, applications to geological and geotechnical problems, and references to the earlier work of Coulomb, von Karman, Love and Biot.

Other selected books covering material relevant to this chapter are the following:

- Atkinson, B. K. (ed.), 1987. *Fracture Mechanics of Rock*, Academic Press, London.
- Evans, B., Wong, T.-F. (eds.), 1992. *Fault Mechanics and Transport Properties of Rocks*, Academic Press, San Diego.
- Paterson, M.S., 1978. *Experimental Rock Deformation — The Brittle Field*, Springer-Verlag, Berlin. (A revised, updated and expanded version by M. S. Paterson and T.-F. Wong is in preparation).
- Scholz, C. H., 1990. *The Mechanics of Earthquakes and Faulting*, Cambridge University Press, Cambridge.
- NRC/NAS, 1996. *Rock Fractures and Fluid Flow*, National Academy Press, Washington, D.C.

A.3. Journals and proceedings

Many of the technical articles that form the basis of this chapter have appeared in *Journal of Geophysical Research* (Solid Earth), *Pure and Applied Geophysics* (PAGEOPH), *Bulletin of the Seismological Society of America* and *International Journal of Rock Mechanics and Mining Science* and in recent Proceedings of the U.S. National Symposium on Rock Mechanics, the North American Rock Mechanics Symposium and the International Congress on Rock Mechanics. Articles on modeling of dynamic rupture and stability of frictional slip systems have also appeared frequently in the *Journal of the Mechanics and Physics of Solids*. A collection of articles on the mechanics of earthquakes appears in *Earthquake Prediction: The Scientific Challenge, Proceedings of the National Academy of Science*, **93**, 3811–3818 (1996). A forthcoming report from the National Academy of Sciences is entitled *Living on a Restless Earth: The Challenge of Earthquake Science* (National Research Council, Committee on the Science of Earthquakes). A recent summary of research on rock deformation appears in an article by S. Karato and T.-F.Wong, entitled 'Rock deformation: ductile and brittle' in the *U. S. National Report to International Union of Geodesy and Geophysics 1991-1994*, Reviews of Geophysics, Supplement, pp. 451–457 (1995).

Selected journal articles are listed below:

- Effects of constitutive behavior on predictions of folding vs. faulting: N. Triantafyllidis and Y. M. Leroy, Stability of a frictional material layer resting on a viscous half-space, *J. Mech. Phys. Solids*, **42**, 51–110, 1994; Stability of a frictional, cohesive layer on a viscous substratum: validity of asymptotic solution and influence of material properties. *J. Geophys. Res.*, **102**, 20551–20570, 1997.
- Constitutive model for the WIPP site: Munson, D. E. Constitutive model of creep in rock salt applied to underground room closure. *Int. J. Rock Mech. Min. Sci.*, **34**, 233–247, 1997.
- Experimental study of AMOCO building marble: Logan, J. M., Hastedt, M., Lehnert, D., Denton, M. A case study of the properties of marble as a building veneer. *Int. J. Rock Mech. & Min. Sci.*, **30**, 1531–1537, 1993.
- Germanovich, L.N., Carter, B.J., Ingraffea, A. R., Dyskin, A. V. and Lee, K. K. Mechanics of 3-D crack growth under compressive loads. In *Rock Mechanics Tools and Techniques, Proc. of 2nd North American Rock Mechanics Symposium* (Edited by M. Aubertin, F. Hassani and H. Mitri), Vol 2, 1151–1160, Balkema, Rotterdam 1996.
- Detournay, E. Coupled thermo-hydro-mechanical processes in rock mechanics, with applications to the petroleum industry, Proc. 8th International Congress on Rock Mechanics, Tokyo (Edited by T. Fuji) Vol. 3, 1061–1068, Balkema, Rotterdam, 1995.

- Discussion of short-duration earthquake slip pulses and references to other articles: Zheng, G., Rice, J. R. Conditions under which velocity-weakening friction allows a self-healing versus a crack-like mode of rupture. *Bulletin of the Seismological Society of America*, **88**, 1466–1483, 1998.
- Cochard, A., Madariaga, R. Complexity of seismicity due to highly rate dependent friction. *J. Geophys. Res.* **101**, 25321–25336, 1996.
- Morrissey, J. W., Rice, J. R. Crack front waves. *J. Mech. Phys. Solids*, **46**, 467–487, 1998.